Many-particle systems. IX. RIP and SHRIMP models

1977 J. Phys. A: Math. Gen. 10 L59

(http://iopscience.iop.org/0305-4470/10/4/002)

J. Phys. A: Math. Gen., Vol. 10, No. 4, 1977. Printed in Great Britain. © 1977

LETTER TO THE EDITOR

Many-particle systems IX. RIP and SHRIMP models

R J M Carr† and H R Post‡

†Department of Mathematics, Bedford College, University of London, London NW1 4NS, UK

‡ Department of History and Philosophy of Science, Chelsea College, University of London, London SW3 6LX, UK

Received 10 February 1977

Abstract. The energy lower bound models HIP and SHIP may be substantially improved for small numbers of fermions by a simple modification to the limiting procedure in their derivation. It is shown that for intermediate values of N the modified SHIP model gives in some cases the best available lower bounds.

We consider a translation invariant system antisymmetric in N particles interacting by pair forces where the ith particle has mass $m_i = m$, $i = 1, 2, \ldots, N$. To obtain the SHIP model we pick out each of the particles $i = 1, 2, \ldots, N$ in turn and allow the mass m_i to become infinite while keeping the remaining $N-1$ particle masses constant§. A modification is to increase m_i but at the same time decrease the masses of the other particles m_j so as to keep $(1/m_i)+[(N-1)/m_j]$ constant. This makes the inequality (6) in Carr and Post (1971, p 668) an equality. The derivation may then proceed as before yielding a lower bound model with the masses m replaced by $(N-1)m/N$ increasing the kinetic energy and thus giving an improved lower bound. A similar modification may also be made to the HIP model of Carr and Post (1968). We call the modified SHIP model the SHRIMP (symmetrized heavy reduced independent many-particle) model and the modified HIP model the RIP (reduced independent particle) model.

The following ordering holds (for all interactions):

$$E_{\mathrm{SHRIMP}} > E_{\mathrm{SHIP}} > E_{\mathrm{HIP}}$$

$$E_{\mathrm{SHRIMP}} > E_{\mathrm{RIP}} > E_{\mathrm{HIP}}.$$

The relation of E_{SHIP} to E_{RIP} is interaction dependent. In three dimensions due to the greater number of degrees of freedom the density of states is greater than in the corresponding one-dimensional problem and the symmetry restriction has less effect in raising the lower bound. We note in several cases that $E_{\mathrm{SHIP}} > E_{\mathrm{RIP}}$ in one dimension but the inequality is reversed in three dimensions.

Some idea of the improvement produced by the reduced mass $(N-1)m/N$ may be obtained from table 1. The notation of Carr and Post (1968, 1971) is used, with

$$E' = \frac{2mEa^2}{\hbar^2} \qquad V' = \frac{2mV_0a^2}{\hbar^2}.$$

The effect in the two-particle case is greater. A striking example is given by table 2.

§ See Carr and Post (1971). Degeneracies due to spin are ignored.

Table 1. Three particles in one dimension, $V' = 50$.

	Square-well interaction	Exponential interaction
E' upper bound	−98·27	−52·88
E'_{SHRIMP}	−123·8	−77·14
E'_{SHIP}	−131·6	−84·54
E'_{RIP}	−135·9	−91·58
E'_{HIP}	−140·1	−97·89

Table 2. Two particles in three dimensions, square-well interaction, $V' = 50$.

E'_0 exact	−22·93
E'_{SHRIMP}	−29·73
E'_{RIP}	−36·52
E'_{SHIP}	−38·59
E'_{HIP}	−42·47

To compare the SHRIMP model with the best available lower-bound methods† we consider the case of Hooke's interaction in one dimension ($V_{ij} = k^2(x_i - x_j)^2$) following Hall (1972). From table 3 it is seen that the OP (one particle) model (Post 1956, called method I in Hall 1972) is best for $N = 2, 3$ and the SHRIMP model for $4 \leqslant N \leqslant 7$. From the energy expressions the Hall (1967) method (method II in Hall 1972) is superior for $N \geqslant 8$. These results demonstrate that the SHRIMP model may in certain cases give the best available lower bound for intermediate values of N, the OP model being best for small N and the Hall (1967) method best for large N. For N sufficiently large the Hall (1967) method is superior for most interactions. The SHRIMP model is superior for sufficiently low density of states.

Table 3. N particles in one dimension, Hooke's interaction‡.

N	Exact $(N^2-1)N^{1/2}$	OP $3(N-1)N^{1/2}$	SHRIMP $2^{-1/2}N^2(N-1)^{1/2}$	Hall (1967) $\frac{1}{2}(N-1)^2(3N)^{1/2}$
2	4·243	4·243	2·828	1·225
3	13·86	10·39	9·000	6·000
4	30·00	18·00	19·60	15·59
5	53·67	26·83	35·36	30·98
6	85·73	36·74	56·92	53·03
7	127·0	47·62	84·87	82·49
8	178·2	59·40	119·7	120·0

‡ Energy in units of $k' = (\hbar^2/2m)^{1/2}k$.

Use has been made of the OP lower bound in nuclear physics (see Brink and Peierls 1968, Humberstone *et al* 1968, Kok *et al* 1968, etc). It is to be hoped that with the reduced mass improvement the N-particle lower-bound model may give sufficiently

† The method of Temple (1928) is excluded because it is dependent on knowledge of the energy of the first excited state.

good results to prove useful in phenomenological calculations involving spatially antisymmetric states especially since the angular momentum considerations of Carr and Post (1971) are applicable.

The authors wish to thank Mr G Horton for many useful discussions.

References

Brink D M and Peierls R E 1968 *Comm. Nucl. Particle Phys.* **2** 92–6
Carr R J M and Post H R 1968 *J. Phys. A: Gen. Phys.* **1** 596–609
—— 1971 *J. Phys. A: Gen. Phys.* **4** 665–78
Hall R L 1967 *Proc. Phys. Soc.* **91** 16–22
—— 1972 *J. Phys. A: Gen. Phys.* **5** 608–11
Humberstone J W, Hall R L and Osborn T A 1968 *Phys. Lett.* **27B** 195–8
Kok L P, Erens G and Van Wageningen R 1968 *Nucl. Phys.* A **122** 684–8
Post H R 1956 *Proc. Phys. Soc.* A **69** 936–8
Temple G 1928 *Proc. R. Soc.* **119** 276–93